Quasiparticle transport in arrays of chaotic cavities

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Abstract. – We find the distribution of transmission eigenvalues in a series of identical junctions between chaotic cavities using the circuit theory of mesoscopic transport. This distribution rapidly approaches the diffusive wire limit as the number of junctions increases, independent of the specific scattering properties of a single junction. The cumulant generating function and the first three cumulants of the charge transfer through the system are obtained both in the normal and in the superconducting state.

Introduction. – Higher-order correlators of current fluctuations in mesoscopic conductors have been studied extensively over the last decade both theoretically [1–7] and experimentally [8–18]. The reason is that they contain, in general, additional information to the usual differential conductance such as higher moments of the transmission eigenvalue distribution, the value of effective charge involved in transport processes, the size of internal energy scales of the system or the correlations intrinsic to the many-body state of entangled systems [19–21]. While the conductance is proportional to the average transmission probability of the structure at low temperatures, the current noise power P_I depends on the second moment of transmission eigenvalue distribution which is characterized by the Fano factor $F = P_I/2eI = [\sum_n T_n(1-T_n)]/\sum_n T_n$. Here e is the electron charge, I is the average current through the sample, and T_n are the transmission eigenvalues. Recent experiments on noise confirmed the theoretical predictions [2,3] on the universal distributions of transmission eigenvalues in a metallic diffusive wire [8, 9] and in an open chaotic cavity [10] with Fano factors $F = 1/3$ and $F = 1/4$, respectively. The crossover from a single cavity to the diffusive wire limit as the number of internal junctions increases was studied experimentally by Oberholzer *et al.* [10] and Song *et al.* [11] recently.

Particle-hole correlations introduced by a superconducting terminal also modify the noise. The low temperature noise of the subgap transport is doubled for tunnel junctions [12] and in diffusive normal wires in contact with a superconductor [13, 14]. The noise in an open cavity is found to be more than two times larger in the superconducting state [15] than in the corresponding normal state junction, in agreement with theoretical predictions [2].

The third correlator contains the first three moments of transmission eigenvalue distribution and is related to the asymmetry of the distribution [22]. In contrast to the current noise which is thermally dominated at temperatures larger than the bias voltage according to the fluctuation-dissipation theorem, the third correlator is in this regime proportional to the current, without the need to correct for the thermal noise. However, higher-order correlators are increasingly more difficult to measure because of the statistical fluctuations [23] and the influence of environment [17, 20]. Recent measurements of the third-order correlations of voltage fluctuations across the nonsuperconducting tunnel junctions by Bomze *et al.* [18] confirmed the Poisson statistics of electron transfer at negligible coupling of the system to environment.

The statistical theory of transport, full counting statistics [24, 25], provides the most detailed description of charge transfer in mesoscopic conductors. The semiclassical cascade approach to higher-order cumulants based on the Boltzmann-Langevin equations has been developed by Nagaev *et al.* [26]. The stochastic path integral theory of full counting statistics was introduced by Pilgram *et al.* [27]. The quantum-mechanical theory of full counting statistics based on the extended Keldysh-Green's function technique [28–30] in the discretized form of the circuit theory [31, 32] was put forward for multiterminal circuits by Nazarov and Bagrets [33]. In this article we use the circuit theory [34, 35] to study the elastic quasiparticle transport in arrays of chaotic cavities focusing on the crossover from a single cavity to the universal limit of a diffusive wire [36] as the number of inner contacts increases. We find the analytical expressions for the distribution of transmission eigenvalues, the cumulant generating function and the first three cumulants both in the normal and in the superconducting state, generalizing the previous results on noise [10] in such a system to all higher-order correlations. The similar finite-size effects on the noise and the third correlator have been studied numerically by Roche and Douçot [37] within an exclusion model. Ballistic to diffusive crossover in metallic conductors with obstacles as a function of increasing disorder has been studied by Macêdo [38] within the scaling theory of transport combined with the circuit theory. The effects of Coulomb interaction on the current and noise in chaotic cavities and diffusive wires have been studied by Golubev *et al.* [39]. The noise in series of junctions has been measured by Oberholzer *et al.* [10] and Song *et al.* [11] recently.

Transport in an array of chaotic cavities. – The system we consider consists of chaotic cavities in series between N identical junctions characterized by N_{ch} transverse channels with transmission eigenvalues $\{T_n\}$. We can neglect the energy dependence of the transmission eigenvalues of the system if the electron dwell time is small with respect to time scales set by the inverse temperature and applied voltage. Also we neglect the charging effects assuming that the conductances of the junctions are much larger than the conductance quantum $2e^2/h$. The quasiparticle distribution function is isotropic between junctions due to chaotic scattering in the cavities. We apply the circuit theory of mesoscopic transport and represent the specific parts of the system by the corresponding discrete circuit elements, as shown in fig. 1. The Greens functions of the leads are denoted as $\check{G}_0(0) \equiv \check{G}_L(0)$ and $\check{G}_N(\chi) \equiv \check{G}_R(\chi)$, while the Greens functions of the internal nodes are $\check{G}_i(\chi)$, $i = 1, \dots, N-1$. The counting field χ can be incorporated through the boundary condition [35] at the right lead according to

$$\check{G}_N(\chi) = e^{-\mathrm{i}(\chi/2)\check{\tau}_K}\ \check{G}_N(0)\ e^{\mathrm{i}(\chi/2)\check{\tau}_K}. \tag{1}$$

Here $\check{G}_{0,N}(0)$ are the bare Greens functions of the Fermi leads in the Keldysh($\bar{\ }$) $\otimes$ Nambu($\hat{\ }$) space, $\bar{\tau}_i$ and $\hat{\sigma}_i$ are the Pauli matrices and $\check{\tau}_K = \bar{\tau}_1\hat{\sigma}_3$. The connection between adjacent

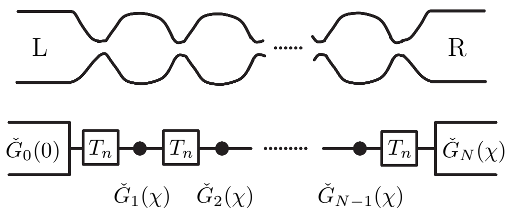

Fig. 1 – An array of chaotic cavities in series between the Fermi leads (top) and the discrete circuit-theory representation of the system (bottom). The leads and the cavities are associated with the corresponding matrix Greens functions $\check{G}_i$. The junctions are characterized by a set of transmission eigenvalues $\{T_n\}$. The matrix current is conserved throughout the circuit.

nodes is described in general by a matrix current [31]

$$\check{I}_{i,i+1} = \frac{2e^2}{h} \sum_{n=1}^{N_{ch}} \frac{2T_n[\check{G}_{i+1}, \check{G}_i]}{4 + T_n(\{\check{G}_{i+1}, \check{G}_i\} - 2)}, \tag{2}$$

which flows from the node i to the node $i+1$. The set of circuit-theory equations for the Greens functions of the internal nodes consists of matrix current conservations $\check{I}_{i,i+1} = const$ and normalization conditions $\check{G}_i^2 = 1$. As shown in ref. [40], we can seek for the solution in the form $\check{G}_i = \check{F}_{iL}(\{\check{G}_0, \check{G}_N\})\, \check{G}_0 + \check{F}_{iR}(\{\check{G}_0, \check{G}_N\})\, \check{G}_N$, where the functions $\check{F}_{iL,R}$ depend only on the anticommutator $\{\check{G}_0, \check{G}_N\}$ and therefore commute with all $\check{G}_j$. As a consequence, the anticommutators $\{\check{G}_i, \check{G}_j\}$ depend only on $\{\check{G}_0, \check{G}_N\}$ and commute with all $\check{G}_k$. We emphasize that the above consideration is independent of the concrete matrix structure of the Greens functions and relies only on the quasiclassical normalization conditions $\check{G}_i^2 = 1$. Because the junctions are identical, the matrix current conservation reduces to [40]

$$\check{G}_i = \frac{\check{G}_{i-1} + \check{G}_{i+1}}{\sqrt{\{\check{G}_{i-1}, \check{G}_{i+1}\} + 2}}. \tag{3}$$

Taking the anticommutator of eq. (3) with $\check{G}_i$ and $\check{G}_{i+1}$, respectively, we find that $\{\check{G}_{i-1}, \check{G}_i\} = \{\check{G}_i, \check{G}_{i+1}\} \equiv \check{\mathcal{G}}'$, for all i. Our aim is to find $\check{\mathcal{G}}'$ in terms of $\check{\mathcal{G}} \equiv \{\check{G}_0, \check{G}_N\}$. Now we take the anticommutator of eq. (3) with $\check{G}_0$ and obtain the following difference equation

$$\check{\gamma}_{i+1} - \check{\mathcal{G}}'\check{\gamma}_i + \check{\gamma}_{i-1} = 0, \tag{4}$$

where $\check{\gamma}_i = \{\check{G}_0, \check{G}_i\}$. After solving eq. (4) with the boundary conditions $\check{\gamma}_0 = 2$ and $\check{\gamma}_N = \check{\mathcal{G}}$, and using that $\check{\gamma}_1 = \check{\mathcal{G}}'$, we find

$$\check{\mathcal{G}}' = [(\check{\mathcal{G}} + \sqrt{\check{\mathcal{G}}^2 - 4})/2]^{1/N} + [(\check{\mathcal{G}} - \sqrt{\check{\mathcal{G}}^2 - 4})/2]^{1/N}. \tag{5}$$

The cumulant generating function $S(\chi)$ of charge transfer through the structure can be obtained as a sum of the actions of the connected pairs of nodes [33]. For identical junctions in series $S(\chi)$ reduces to the contribution of a single junction multiplied by N:

$$S(\chi) = -\frac{t_0}{2h} \int d\mathcal{E}\ \mathrm{tr}[\tilde{S}(\chi)], \qquad \text{with} \qquad \tilde{S}(\chi) = N \sum_{n=1}^{N_{ch}} \ln[1 + T_n(\check{\mathcal{G}}' - 2)/4]. \tag{6}$$

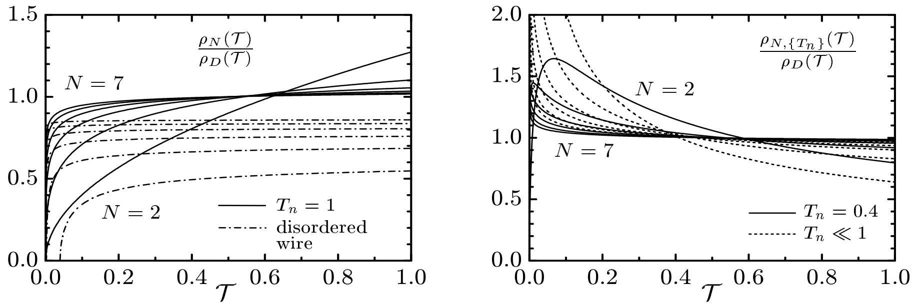

Fig. 2 – Distribution of transmission eigenvalues for N open contacts in series (left panel) and for contacts of lower transparency $T_n = 0.4$ and $T_n \ll 1$ (right panel), normalized to the transmission distribution of a diffusive wire $\rho_D(\mathcal{T})$. Ballistic to diffusive crossover in a metallic disordered wire is shown for comparison (left panel, dash-dotted curves) as a function of increasing disorder $L/l = N-1$.

Here t_0 is the total measurement time which is much larger than the characteristic time scale on which the current fluctuations are correlated. For a large number of junctions, $N \gg 1$, the cumulant generating function given by eq. (6) approaches the universal limit of a diffusive wire [25,32] $S(\chi) = (-t_0\tilde{g}/8h)\int \mathrm{d}\mathcal{E}\ \mathrm{tr}[\mathrm{arccosh}^2(\check{\mathcal{G}}/2)]$, which does not depend on the specific scattering properties $\{T_n\}$ of a single junction, the shape of the conductor or the impurity distribution [34,36]. Here $\tilde{g} = (\sum_n T_n)/N$ is the total conductance of a wire in units of $2e^2/h$.

The distribution of transmission eigenvalues of a composite junction $\rho_{N,\{T_n\}}(\mathcal{T})$ is directly related to the cumulant generating function by [32]

$$\rho_{N,\{T_n\}}(\mathcal{T}) = \frac{1}{\pi\mathcal{T}^2}\,\mathrm{Im}\Big(\frac{\partial\tilde{S}(\zeta)}{\partial\zeta}\Big|_{\zeta=-1/\mathcal{T}-\mathrm{i}\,0}\Big), \tag{7}$$

where $\zeta = (\check{\mathcal{G}} - 2)/4$. From eqs. (5), (6) and (7) we find

$$\rho_{N,\{T_n\}}(\mathcal{T}) = \rho_D(\mathcal{T})\,\frac{\sin(\pi/N)}{\pi/N} \times \left\langle \frac{4T_n[(b^{1/N}+b^{-1/N})(2-T_n)+2T_n\cos(\pi/N)]}{[(b^{1/N}+b^{-1/N})(2-T_n)+2T_n\cos(\pi/N)]^2 - 4(1-T_n)(b^{1/N}-b^{-1/N})^2} \right\rangle. \tag{8}$$

Here $\rho_D(\mathcal{T}) = (\tilde{g}/2)(1/\mathcal{T}\sqrt{1-\mathcal{T}})$ is the transmission distribution of a diffusive wire, $b = (1+\sqrt{1-\mathcal{T}})^2/\mathcal{T}$, and $\langle\cdots\rangle = (\sum_n \cdots)/(\sum_n T_n)$ denotes the averaging over the transmission eigenvalues of a single junction. For N open contacts in series ($T_n = 1$) eq. (8) reduces to

$$\rho_N(\mathcal{T}) = \rho_D(\mathcal{T})\,\frac{\sin(\pi/N)}{\pi/N}\,\frac{4\mathcal{T}^{1/N}}{(1+\sqrt{1-\mathcal{T}})^{2/N}+(1-\sqrt{1-\mathcal{T}})^{2/N}+2\mathcal{T}^{1/N}\cos(\pi/N)}. \tag{9}$$

The crossover from a single cavity to the diffusive regime as the number of junctions N increases is shown in fig. 2 for open contacts (left panel) and for contacts of lower transparency $T_n = 0.4$ and $T_n \ll 1$ (right panel). The transmission distribution of a metallic disordered wire of the length L and the mean free path l is shown for comparison (dash-dotted curves in fig. 2; cf. ref. [3]).

The transmission eigenvalue distributions given by eqs. (8) and (9) can be probed experimentally by measuring higher-order correlators of current fluctuations across the junction at low temperatures. The first three moments of charge transport statistics are related to the average current, the current noise power and the third correlator according to

$I = \mathrm{i}(e/t_0)\partial_\chi S|_{\chi=0}$, $P_I = (2e^2/t_0)\partial_\chi^2 S|_{\chi=0}$, and $C_I = -\mathrm{i}(e^3/t_0)\partial_\chi^3 S|_{\chi=0}$, respectively. In the linear regime, which we consider here, the current is proportional to the bias voltage with conductance given by $\tilde{g} = (\sum_n T_n)/N = \int_0^1 \mathrm{d}\mathcal{T}\rho_{N,\{T_n\}}(\mathcal{T})\ \mathcal{T}$ in units of $2e^2/h$. At temperatures much lower than the bias voltage, the current noise power and the third correlator are linear in the current, with the slopes given by the Fano factor $F = \partial P_I/\partial(2eI) = (1/\tilde{g})\int_0^1 \mathrm{d}\mathcal{T}\rho_{N,\{T_n\}}(\mathcal{T})\mathcal{T}(1-\mathcal{T})$ and the "skewness" $C = \partial C_I/\partial(e^2 I) = (1/\tilde{g})\int_0^1 \mathrm{d}\mathcal{T}\rho_{N,\{T_n\}}(\mathcal{T}) \times \mathcal{T}(1-\mathcal{T})(1-2\mathcal{T})$, respectively. For the normal-state junction the two parameters are given by

$$F = \frac{1}{3}\left(1 + \frac{2 - 3\langle T_n^2\rangle}{N^2}\right) \tag{10}$$

and

$$C = \frac{1}{15}\left(1 + \frac{5(2-3\langle T_n^2\rangle)}{N^2} + \frac{4 - 30\langle T_n^2(1-T_n)\rangle}{N^4}\right). \tag{11}$$

The Fano factor given by eq. (10) coincides with the result previously obtained within Boltzmann-Langevin approach which takes into account both cavity noise and partition noise at the contacts and was confirmed experimentally for up to three open contacts in series [10]. The sign of C is related to the asymmetry of transmission distribution [22], being negative (positive) when the weight of the distribution is shifted towards open (closed) transmission channels. Equation (11) shows that closed channels prevail in the composite junction for $N > 2$ even for completely open inner contacts, in agreement with eq. (9).

Now we focus on the junction sandwiched between a normal-metal and a superconductor in the coherent regime in which we can neglect the particle-hole dephasing ($E_{th} \gg |eV|, k_B T, \Delta$ where E_{th} is the inverse dwell time). At temperatures and bias voltages smaller than the superconducting gap Δ, the transport properties can be obtained by integrating the Andreev reflection probability $R_A = \mathcal{T}^2/(2-\mathcal{T})^2$ over the transmission distribution [2,40] and correcting for the effective charge $e^* = 2e$. The conductance, the Fano factor and the skewness are given by $\tilde{g}_S = \int_0^1 \mathrm{d}\mathcal{T}\ \rho_{N,\{T_n\}}(\mathcal{T})\ R_A$ (in units of $4e^2/h$), $F_S = \partial P_I/\partial(2e^* I) = (1/\tilde{g}_S)\int_0^1 \mathrm{d}\mathcal{T}\rho_{N,\{T_n\}}(\mathcal{T})R_A(1-R_A)$, and $C_S = \partial C_I/\partial(e^{*2} I) = (1/\tilde{g}_S)\int_0^1 \mathrm{d}\mathcal{T}\rho_{N,\{T_n\}}(\mathcal{T})R_A(1-R_A)(1-2R_A)$, respectively. For N open contacts in series we find

$$\tilde{g}_S = \frac{N_{ch}}{2N}\frac{1}{\cos^2(\pi/4N)}, \qquad F_S = \frac{1}{3}\Big[1 - \frac{1}{4N^2}\Big(\frac{3}{\cos^2(\pi/4N)} - 2\Big)\Big], \tag{12}$$

and

$$C_S = \frac{1}{15}\Big[1 - \frac{5}{4N^2}\Big(\frac{3}{\cos^2(\pi/4N)} - 2\Big) + \frac{1}{8N^4}\Big(2 + 15\frac{\sin^2(\pi/4N)}{\cos^4(\pi/4N)}\Big)\Big]. \tag{13}$$

The skewness C_S given by eq. (13) is positive for $N > 1$ which indicates that the asymmetry of the Andreev reflection distribution $\rho_{NA}(R_A) = \rho_N(\mathcal{T})d\mathcal{T}/dR_A$ is in favor of closed Andreev channels [40]. For the more general case of N contacts characterized by transmission eigenvalues $\{T_n\}$ we obtain $\tilde{g}_S = (\sum_n \alpha_n)/N$ and $F_S = 1 - (\sum_n \alpha_n\beta_n)/(\sum_n \alpha_n)$, where $\alpha_n = \sqrt{R_n^A}[\sqrt{R_n^A} + \cos(\pi/2N)]/[1 + \sqrt{R_n^A}\cos(\pi/2N)]^2$, $\beta_n = 2/3 - (1/6N^2)\{1 - 6\alpha_n + 3\sqrt{R_n^A}/[\sqrt{R_n^A} + \cos(\pi/2N)]\}$, and $R_n^A = T_n^2/(2-T_n)^2$. Fano factors and skewnesses for the normal-state and superconducting junctions are shown in fig. 3 as a function of the number of contacts in series and for different contact transparencies. It is interesting to note that in the coherent superconducting regime, which we consider here, the higher-order correlators satisfy the approximate scaling relations $F_S(N) \approx F(2N)$ and $C_S(N) \approx C(2N)$ which are exact for incoherent Andreev transport [41]. For large N this results in the full reentrance of transport properties of a diffusive wire in contact with a superconductor [30] as a function of the particle-hole coherence.

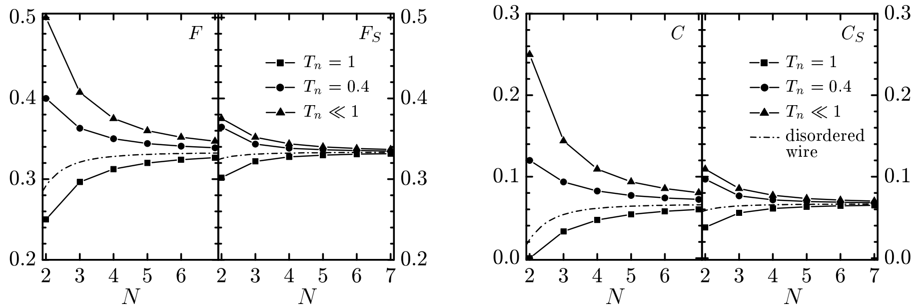

Fig. 3 – The Fano factor F (left panel) and the skewness C (right panel) as a function of the number of contacts in series N, shown for different contact transparencies. The corresponding distributions of transmission eigenvalues of the composite junctions are shown in fig. 2. The Fano factors F_S and the skewnesses C_S (normalized by $e^* = 2e$) of the superconducting junctions are given for comparison. Ballistic to diffusive crossover in a disordered wire ($L/l = N - 1$) is shown by dash-dotted curves.

Conclusion. – We have studied the transport properties of several chaotic cavities in series using the circuit theory of mesoscopic transport. We obtained the analytical expression for the distribution of transmission eigenvalues of the composite junction as a function of the number of contacts and the scattering properties of a single contact. This distribution generalizes the previous results on noise in such a system [10] to all higher-order cumulants. As an example we found the first three cumulants of the charge transfer statistics both for the normal-state junction and in the case when one lead is superconducting. The sign of the third cumulant at high bias can be used to probe the asymmetry of the transmission eigenvalue distribution: it is negative (positive) when the weight of the distribution is more on open (closed) transport channels. As the number of contacts increases, all transport properties approach the universal limit of a diffusive wire [36]. While the crossover from a few cavities to the diffusive-wire limit has already been studied through the noise in the normal state [10,11], experimental investigations of either higher-order correlators or cavities in contact with a superconductor are still to come.

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